

Fermionic T-duality and momenta noncommutativity *

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Abstract

In this article we establish the relationship between fermionic T-duality and momenta noncommutativity. This is extension of known relation between bosonic T-duality and coordinate noncommutativity. The case of open string propagating in background of the type IIB superstring theory has been considered. We perform T-duality with respect to the fermionic variables instead to the bosonic ones. We also choose Dirichlet boundary conditions at the string endpoints, which lead to the momenta noncommutativity, instead Neumann ones which lead to the coordinates noncommutativity. Finally, we establish the main result of the article that momenta noncommutativity parameters are just fermionic T-dual fields.

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1 Introduction

Two theories that are dual to one another can be viewed as being physically identical [1]. An important kind of duality is so called T-duality, where T stands for target space-time. This means that we can switch the target space with its dual without losing the physical content of the theory.

When the open string endpoints are attached to D-brane, its world-volume becomes noncommutative manifold [2, 3, 4, 5, 6]. The noncommutativity parameter is proportional to the Neveu-Schwarz antisymmetric field $B_{\mu\nu}$, while in the supersymmetric case the noncommutative parameters are proportional to the Ω odd parts of NS-R field, $\Psi^\alpha_{-\mu}$, and

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R-R field strength, $F_s^{\alpha\beta}$. The noncommutative (super)space is starting point in studying properties of the noncommutative (super) Yang-Mills theories [7].

Recently a new kind of T-duality was discovered, the fermionic T-duality [8]. It consists in certain non-local redefinitions of the fermionic variables of the superstring mapping a supersymmetric background to another supersymmetric background. Technically fermionic T-duality is similar to the bosonic one, except that dualization is performed along fermionic directions, θ^α and $\bar{\theta}^\alpha$. Ref.[8] also shows that T-duality maps gluon scattering amplitudes in the original theory to Wilson loops in the dual theory. They also investigated connection between "dual conformal symmetry" and integrability. The articles [9], focussing more on integrability, deal with fermionic T-duality also, using Green-Schwarz string on $AdS_5 \times S^5$. From slightly different point of view most of the results of the Ref.[8] have been obtained.

The present article is motivated by the fact that for the specific solution of the boundary conditions some of the bosonic T-dual background fields coincide with noncommutativity parameters [10, 11]. In these articles type IIB superstring theory in pure spinor formulation has been considered. Performing Buscher T-duality [12] along all bosonic directions x^μ , the background fields of the T-dual theory have been found. On the other hand, consequences of the particular boundary conditions at the open string endpoints have been investigated: the Neumann boundary conditions for bosonic coordinates and preserving half of the initial $N = 2$ supersymmetry for fermionic ones. It turned out that coordinates noncommutativity parameters are the bosonic T-dual fields. So, the particular choice of duality (along bosonic directions) corresponds to the particular choice of boundary conditions. In the present article we are looking for such boundary conditions which produce noncommutativity parameters equal to the fermionic T-dual background fields.

The article is organized in the following way. First, we introduce the action of the pure spinor formulation for type IIB superstring theory keeping quadratic terms. Then, we perform canonical analysis in the light-cone coordinates. Because of reparameterization invariance we can take any timelike or lightlike coordinate as evolution parameter. For lightlike evolution parameter the Lagrangian is linear in velocities, and there are primary constraints which we will use as suitable introduced currents. There are two cases for consideration: 1) $\tau \rightarrow \sigma_-$ and $\sigma \rightarrow \sigma_+$ and 2) $\tau \rightarrow \sigma_+$ and $\sigma \rightarrow -\sigma_-$. Canonical Hamiltonian with timelike evolution parameter can be written in the Sugawara form of the currents.

In the case of open string action principle, besides equations of motion, produces boundary conditions. Choosing Dirichlet boundary conditions and treating them as canonical constraints [5, 6], we obtain the initial coordinates and momenta in terms of the effective ones, which are odd under world-sheet parity transformation $\Omega : \sigma \rightarrow -\sigma$. It turns out that momenta are noncommutative, while the coordinates are commutative. The source

of noncommutativity is the presence of the effective coordinates in the solution for initial momenta. The noncommutativity parameters are fermionic T-dual background fields.

At the end we give some concluding remarks.

2 Type IIB superstring and fermionic T-duality

In this section we will introduce the action of type IIB superstring theory in pure spinor formulation and perform fermionic T-duality [8].

The action of type IIB superstring theory in pure spinor formulation (up to the quadratic terms [13, 14, 15, 10, 11] and neglecting ghost terms as in Ref.[14]) is of the form

$$S = \kappa \int_{\Sigma} d^2 \xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu + \int_{\Sigma} d^2 \xi \left[-\pi_\alpha \partial_- (\theta^\alpha + \Psi_\mu^\alpha x^\mu) + \partial_+ (\bar{\theta}^\alpha + \bar{\Psi}_\mu^\alpha x^\mu) \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar{\pi}_\beta \right], \quad (2.1)$$

where the world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau, \xi^1 = \sigma)$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Superspace is spanned by bosonic coordinates x^μ ($\mu = 0, 1, 2, \dots, 9$) and fermionic ones, θ^α and $\bar{\theta}^\alpha$ ($\alpha = 1, 2, \dots, 16$). The variables π_α and $\bar{\pi}_\alpha$ are canonically conjugated momenta to θ^α and $\bar{\theta}^\alpha$, respectively. All spinors are Majorana-Weyl ones and $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}$.

On the equations of motion for fermionic momenta π_α and $\bar{\pi}_\alpha$ we obtain

$$\pi_\alpha = -\frac{1}{2} \partial_+ \bar{\eta}_\alpha, \quad \bar{\pi}_\alpha = \frac{1}{2} \partial_- \eta_\alpha, \quad (2.2)$$

where we introduce useful notation

$$\eta_\alpha \equiv 4\kappa (F^{-1})_{\alpha\beta} (\theta^\beta + \Psi_\mu^\beta x^\mu), \quad \bar{\eta}_\alpha \equiv 4\kappa (\bar{\theta}^\beta + \bar{\Psi}_\mu^\beta x^\mu) (F^{-1})_{\beta\alpha}. \quad (2.3)$$

Using these relations the action gets the form

$$S(\partial_\pm x, \partial_- \theta, \partial_+ \bar{\theta}) = \kappa \int_{\Sigma} d^2 \xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu + 2\kappa \int_{\Sigma} d^2 \xi \partial_+ (\bar{\theta}^\alpha + \bar{\Psi}_\mu^\alpha x^\mu) (F^{-1})_{\alpha\beta} \partial_- (\theta^\beta + \Psi_\nu^\beta x^\nu). \quad (2.4)$$

Now we will perform fermionic T-duality presented in Ref.[8]. We suppose that the action has a global shift symmetry in θ^α and $\bar{\theta}^\alpha$ directions. So, we introduce gauge fields (v_+^α, v_-^α) and $(\bar{v}_+^\alpha, \bar{v}_-^\alpha)$ and make a change in the action

$$\partial_- \theta^\alpha \rightarrow D_- \theta^\alpha \equiv \partial_- \theta^\alpha + v_-^\alpha, \quad \partial_+ \bar{\theta}^\alpha \rightarrow D_+ \bar{\theta}^\alpha \equiv \partial_+ \bar{\theta}^\alpha + \bar{v}_+^\alpha. \quad (2.5)$$

In addition we introduce the Lagrange multipliers ϑ_α and $\bar{\vartheta}_\alpha$ which will impose that field strengths of gauge fields v_\pm^α and \bar{v}_\pm^α vanish

$$S_{gauge}(\vartheta, v_\pm, \bar{\vartheta}, \bar{v}_\pm) = \frac{1}{2}\kappa \int_\Sigma d^2\xi \bar{\vartheta}_\alpha (\partial_+ v_-^\alpha - \partial_- v_+^\alpha) + \frac{1}{2}\kappa \int_\Sigma d^2\xi (\partial_+ \bar{v}_-^\alpha - \partial_- \bar{v}_+^\alpha) \vartheta_\alpha, \quad (2.6)$$

and the full action is of the form

$$S^*(x, \theta, \bar{\theta}, \vartheta, \bar{\vartheta}, v_\pm, \bar{v}_\pm) = S(\partial_\pm x, D_- \theta, D_+ \bar{\theta}) + S_{gauge}(\vartheta, \bar{\vartheta}, v_\pm, \bar{v}_\pm). \quad (2.7)$$

If we vary with respect to the Lagrange multipliers ϑ_α and $\bar{\vartheta}_\alpha$ we obtain $\partial_+ v_-^\alpha - \partial_- v_+^\alpha = 0$ and $\partial_+ \bar{v}_-^\alpha - \partial_- \bar{v}_+^\alpha = 0$ which gives

$$\bar{v}_\pm^\alpha = \partial_\pm \bar{\theta}^\alpha, \quad v_\pm^\alpha = \partial_\pm \theta^\alpha. \quad (2.8)$$

Substituting these expression in (2.7) we obtain the initial action (2.4).

Now we can fix θ^α and $\bar{\theta}^\alpha$ to zero and obtain the action quadratic in the fields v_\pm and \bar{v}_\pm

$$\begin{aligned} S^* &= \kappa \int_\Sigma d^2\xi \partial_+ x^\mu \left[\Pi_{+\mu\nu} + 2\bar{\Psi}_\mu^\alpha (F^{-1})_{\alpha\beta} \Psi_\nu^\beta \right] \partial_- x^\nu \\ &+ 2\kappa \int_\Sigma \left[\bar{v}_+^\alpha (F^{-1})_{\alpha\beta} v_-^\beta + \bar{v}_+^\alpha (F^{-1})_{\alpha\beta} \Psi_\nu^\beta \partial_- x^\nu + \partial_+ x^\mu \bar{\Psi}_\mu^\alpha (F^{-1})_{\alpha\beta} v_-^\beta \right] \\ &+ \frac{1}{2}\kappa \int_\Sigma d^2\xi \bar{\vartheta}_\alpha (\partial_+ v_-^\alpha - \partial_- v_+^\alpha) + \frac{1}{2}\kappa \int_\Sigma d^2\xi (\partial_+ \bar{v}_-^\alpha - \partial_- \bar{v}_+^\alpha) \vartheta_\alpha, \end{aligned} \quad (2.9)$$

which can be integrated out classically. On the equations of motion for v_\pm and \bar{v}_\pm we obtain, respectively

$$\partial_- \bar{\vartheta}_\alpha = 0, \quad \bar{v}_+^\alpha = \frac{1}{4} \partial_+ \bar{\vartheta}_\beta F^{\beta\alpha} - \partial_+ x^\mu \bar{\Psi}_\mu^\alpha, \quad (2.10)$$

$$\partial_+ \vartheta_\alpha = 0, \quad v_-^\alpha = -\frac{1}{4} F^{\alpha\beta} \partial_- \vartheta_\beta - \Psi_\mu^\alpha \partial_- x^\mu. \quad (2.11)$$

Substituting these expression in the action S^* we obtain the dual action

$$\begin{aligned} {}^*S(\partial_\pm x, \partial_- \vartheta, \partial_+ \bar{\vartheta}) &= \kappa \int_\Sigma d^2\xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu, \\ &+ \frac{\kappa}{8} \int_\Sigma d^2\xi \left[\partial_+ \bar{\vartheta}_\alpha F^{\alpha\beta} \partial_- \vartheta_\beta - 4\partial_+ x^\mu \bar{\Psi}_\mu^\alpha \partial_- \vartheta_\alpha + 4\partial_+ \bar{\vartheta}_\alpha \Psi_\mu^\alpha \partial_- x^\mu \right], \end{aligned} \quad (2.12)$$

from which we read the dual background fields (denoted by stars)

$${}^*B_{\mu\nu} = B_{\mu\nu} + [(\bar{\Psi} F^{-1} \Psi)_{\mu\nu} - (\bar{\Psi} F^{-1} \Psi)_{\nu\mu}], \quad {}^*G_{\mu\nu} = G_{\mu\nu} + 2[(\bar{\Psi} F^{-1} \Psi)_{\mu\nu} + (\bar{\Psi} F^{-1} \Psi)_{\nu\mu}], \quad (2.13)$$

$${}^*\Psi_{\alpha\mu} = 4(F^{-1}\Psi)_{\alpha\mu}, \quad {}^*\bar{\Psi}_{\mu\alpha} = -4(\bar{\Psi} F^{-1})_{\mu\alpha}, \quad (2.14)$$

$${}^*F_{\alpha\beta} = 16(F^{-1})_{\alpha\beta}. \quad (2.15)$$

Let us note that two successive dualizations give the initial background fields.

3 Canonical structure of the theory

The main technical problem is to perform complete consistency procedure for the constraints because it has infinite many steps. So, it is useful to find such basic variables (currents), which Poisson brackets with Hamiltonian are as simple as possible. Following the idea of Ref.[16], we can obtain these currents as canonical constraints when lightlike direction is evolution parameter. It turns that they are good basis for all canonical super-variables, and that they have simple Poisson brackets as well with Hamiltonian as among them.

3.1 Canonical analysis with light-like evolution parameter

Because of world sheet reparametrization invariance, any timelike or lightlike coordinate could be chosen as evolution parameter. The action (2.4) is linear in derivatives with respect to the light-cone coordinates ∂_{\pm} . So, in order to get some canonical constraints, we have two possibilities: 1) $\sigma_- \rightarrow \tau$ and $\sigma_+ \rightarrow \sigma$, and 2) $\sigma_+ \rightarrow \tau$ and $\sigma_- \rightarrow -\sigma$, where $\sigma_{\pm} = \frac{1}{2}(\tau \pm \sigma)$.

In the first case, $\sigma_- \rightarrow \tau$ and $\sigma_+ \rightarrow \sigma$, the world-sheet action gets the form

$$S = 2\kappa \int_{\Sigma} d^2\xi \left[x'^{\mu} \Pi_{+\mu\nu} \dot{x}^{\nu} + 2(\bar{\theta}'^{\alpha} + \bar{\Psi}_{\mu}^{\alpha} x'^{\mu})(F^{-1})_{\alpha\beta}(\dot{\theta}^{\beta} + \Psi_{\nu}^{\beta} \dot{x}^{\nu}) \right]. \quad (3.1)$$

The canonical momenta conjugated to the variables x^{μ} , θ^{α} and $\bar{\theta}^{\alpha}$

$$\pi_{\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^{\mu}} = 2\kappa \left[-\Pi_{-\mu\nu} x'^{\nu} + \frac{1}{2\kappa} \bar{\eta}'_{\alpha} \Psi_{\mu}^{\alpha} \right], \quad (3.2)$$

$$\pi_{\alpha} = \frac{\partial_L \mathcal{L}}{\partial \dot{\theta}^{\alpha}} = -\bar{\eta}'_{\alpha}, \quad \bar{\pi}_{\alpha} = \frac{\partial_L \mathcal{L}}{\partial \dot{\bar{\theta}}^{\alpha}} = 0, \quad (3.3)$$

do not depend on the τ -derivatives, and consequently, there are primary constraints

$$J_{-\mu} = j_{-\mu} - \bar{\eta}'_{\alpha} \Psi_{\mu}^{\alpha}, \quad J_{-\alpha} = \pi_{\alpha} + \bar{\eta}'_{\alpha}, \quad \bar{J}_{-\alpha} = \bar{\pi}_{\alpha}, \quad (3.4)$$

where we introduce

$$j_{\pm\mu} = \pi_{\mu} + 2\kappa \Pi_{\pm\mu\nu} x'^{\nu}. \quad (3.5)$$

If we use the notation $J_{-A} = (J_{-\mu}, J_{-\alpha}, \bar{J}_{-\alpha})$ and the basic Poisson algebra

$$\{x^{\mu}(\sigma), \pi_{\nu}(\bar{\sigma})\} = \delta^{\mu}_{\nu} \delta(\sigma - \bar{\sigma}), \quad \{\theta^{\alpha}(\sigma), \pi_{\beta}(\bar{\sigma})\} = \{\bar{\theta}^{\alpha}(\sigma), \bar{\pi}_{\beta}(\bar{\sigma})\} = -\delta^{\alpha}_{\beta} \delta(\sigma - \bar{\sigma}), \quad (3.6)$$

algebra of the constraints gets the form

$$\{J_{-A}, J_{-B}\} = -2\kappa^* G_{AB} \delta', \quad (3.7)$$

where

$${}^*G_{AB} = \begin{pmatrix} {}^*G_{\mu\nu} & \frac{1}{2}{}^*\bar{\Psi}_{\mu\gamma} & \frac{1}{2}({}^*\Psi^T)_{\mu\delta} \\ \frac{1}{2}({}^*\bar{\Psi}^T)_{\alpha\nu} & 0 & \frac{1}{8}({}^*F^T)_{\alpha\delta} \\ \frac{1}{2}{}^*\Psi_{\beta\nu} & -\frac{1}{8}{}^*F_{\beta\gamma} & 0 \end{pmatrix}. \quad (3.8)$$

Let us note that ${}^*G_{AB}$ obeys graded symmetrization rule ${}^*G_{AB} = (-)^{AB}{}^*G_{BA}$.

In the second case, $\sigma_+ \rightarrow \tau$ and $\sigma_- \rightarrow -\sigma$, the action is of the form

$$S = -2\kappa \int_{\Sigma} d^2\xi \left[\dot{x}^\mu \Pi_{+\mu\nu} x'^\nu + 2(\dot{\theta}^\alpha + \bar{\Psi}_\mu^\alpha \dot{x}^\mu)(F^{-1})_{\alpha\beta}(\theta'^\beta + \Psi_\nu^\beta x'^\nu) \right]. \quad (3.9)$$

Similarly, we obtain primary constraints

$$J_{+\mu} = j_{+\mu} + \bar{\Psi}_\mu^\alpha \eta'_\alpha, \quad J_{+\alpha} = \pi_\alpha, \quad \bar{J}_{+\alpha} = \bar{\pi}_\alpha + \eta'_\alpha, \quad (3.10)$$

where the corresponding algebra of the constraints $J_{+A} = (J_{+\mu}, J_{+\alpha}, \bar{J}_{+\alpha})$ is

$$\{J_{+A}, J_{+B}\} = 2\kappa {}^*G_{AB} \delta', \quad (3.11)$$

with the same coefficient as in the first case.

It is easy to check that

$$\{J_{+A}, J_{-B}\} = 0, \quad (3.12)$$

so that we obtain two independent Abelian Kac-Moody algebras

$$\{J_{\pm A}, J_{\pm B}\} = \pm 2\kappa {}^*G_{AB} \delta', \quad \{J_{\pm A}, J_{\mp B}\} = 0. \quad (3.13)$$

Note that the algebra of the constraints closes on the fermionic T-dual background fields (2.13)-(2.15) (except ${}^*B_{\mu\nu}$).

Because the action is linear in time derivative in both cases, the canonical Hamiltonian density is zero, $\mathcal{H}_c = 0$, and the total Hamiltonian takes the form

$$H_{T\pm} = \int d\sigma \mathcal{H}_{T\pm} = \int d\sigma \lambda_\pm^A J_{\pm A}, \quad (3.14)$$

where λ^A are Lagrange multipliers. With the help of (3.13) it is easy to check that

$$\dot{J}_{\pm A} = \{J_{\pm A}, H_{T\pm}\} = \mp 2\kappa {}^*G_{AB} \lambda_\pm^B. \quad (3.15)$$

Consequently, there are no more constraints and for $s \det {}^*G_{AB} \sim \frac{\det {}^*G}{\det {}^*F^2} \neq 0$, all constraints, except the zero modes, are of the second class.

3.2 Canonical structure with time like evolution parameter – From Kac-Moody to Virasoro algebra

Following reasons of Ref.[16] we are going to formulate canonical structure with time-like evolution parameter $\tau = \xi^0$ using the structure with light-like ones $\tau = \sigma_+$ and $\tau = \sigma_-$. We construct energy-momentum tensor components in Sugawara form as bilinear combination of the currents $J_{\pm A}$

$$T_{\pm} = \mp \frac{1}{4\kappa} J_{\pm A} (*G^{-1})^{AB} J_{\pm B}, \quad (3.16)$$

where

$$(*G^{-1})^{AB} = \begin{pmatrix} G^{\mu\nu} & -\Psi^{\mu\gamma} & -\bar{\Psi}^{\mu\delta} \\ \Psi^{\alpha\nu} & -\Psi_{\rho}^{\alpha}\Psi^{\rho\gamma} & -\frac{1}{2}(F^{\alpha\delta} + 2\Psi_{\rho}^{\alpha}\bar{\Psi}^{\rho\delta}) \\ \bar{\Psi}^{\beta\nu} & \frac{1}{2}\left((F^T)^{\beta\gamma} - 2\bar{\Psi}_{\rho}^{\beta}\Psi^{\rho\gamma}\right) & -\bar{\Psi}_{\rho}^{\beta}\bar{\Psi}^{\rho\delta} \end{pmatrix}, \quad (3.17)$$

is inverse of supermatrix $*G_{AB}$. Note that the currents $J_{\pm A}$, which was canonical constraints for lightlike evolution parameter, are not canonical constraints for timelike evolution parameter. Here, the canonical constraints are only energy-momentum tensor components. They satisfy two independent Virasoro algebras

$$\{T_{\pm}(\sigma), T_{\pm}(\bar{\sigma})\} = -[T_{\pm}(\sigma) + T_{\pm}(\bar{\sigma})]\delta', \quad \{T_{\pm}(\sigma), T_{\mp}(\bar{\sigma})\} = 0, \quad (3.18)$$

which are equivalent to the algebra of world-sheet diffeomorphisms. The Hamiltonian for $\tau = \xi^0$ is given by

$$H_c = \int d\sigma \mathcal{H}_c, \quad \mathcal{H}_c = T_- - T_+. \quad (3.19)$$

By straightforward calculation we can prove

$$\{H_c, J_{\pm A}\} = \mp J'_{\pm A}. \quad (3.20)$$

Using (3.17) we can obtain the expressions of energy-momentum tensors in terms of the components

$$\begin{aligned} T_{\pm} = & \mp \frac{1}{4\kappa} G^{\mu\nu} J_{\pm\mu} J_{\pm\nu} \mp \frac{1}{2\kappa} J_{\pm\alpha} \Psi^{\alpha\mu} J_{\pm\mu} \mp \frac{1}{2\kappa} \bar{J}_{\pm\alpha} \bar{\Psi}^{\alpha\mu} J_{\pm\mu} \\ & \pm \frac{1}{4\kappa} J_{\pm\alpha} \Psi^{\alpha}\Psi^{\beta} J_{\pm\beta} \pm \frac{1}{4\kappa} J_{\pm\alpha} \left(F^{\alpha\beta} + 2\Psi^{\alpha}\bar{\Psi}^{\beta}\right) \bar{J}_{\pm\beta} \pm \frac{1}{4\kappa} \bar{J}_{\pm\alpha} \bar{\Psi}^{\alpha}\bar{\Psi}^{\beta} \bar{J}_{\pm\beta}. \end{aligned} \quad (3.21)$$

We can check that our construction is equivalent to that of Refs.[15, 11] obtained by prime calculation. Here we used the relation between currents $J_{\pm A}$ and the current $I_{\pm\mu}$ introduced in Refs.[15, 11]

$$I_{\pm\mu} = J_{\pm\mu} + J_{\pm\alpha} \Psi_{\mu}^{\alpha} - \bar{\Psi}_{\mu}^{\alpha} \bar{J}_{\pm\alpha}. \quad (3.22)$$

The currents $\star J_{\pm}^A$, where index is raised by $(\star G^{-1})^{AB}$, are of the form

$$\star J_{\pm}^A \equiv (\star G^{-1})^{AB} J_{\pm B} = \begin{pmatrix} \star J_{\pm}^{\mu} \\ \star J_{\pm}^{\alpha} \\ \star \bar{J}_{\pm}^{\beta} \end{pmatrix} = \begin{pmatrix} J_{\pm}^{\mu} - \Psi^{\mu\alpha} J_{\pm\alpha} - \bar{\Psi}^{\mu\alpha} \bar{J}_{\pm\alpha} \\ \Psi^{\alpha\nu} J_{\pm\nu} - \bar{\Psi}_{\mu}^{\alpha} \Psi^{\beta\mu} J_{\pm\beta} - \frac{1}{2}(F^{\alpha\beta} + 2\Psi_{\mu}^{\alpha} \bar{\Psi}^{\beta\mu}) \bar{J}_{\pm\beta} \\ \bar{\Psi}^{\beta\nu} J_{\pm\nu} + \frac{1}{2}(F^{\gamma\beta} - 2\bar{\Psi}^{\beta\mu} \Psi_{\mu}^{\gamma}) J_{\pm\gamma} - \bar{\Psi}^{\beta\mu} \bar{\Psi}_{\mu}^{\gamma} \bar{J}_{\pm\gamma} \end{pmatrix}. \quad (3.23)$$

Let us note that $\star J_{\pm}^{\mu} = G^{\mu\nu} I_{\pm\nu}$.

4 Boundary conditions as a canonical constraints

In this section we will look for such solution of the boundary conditions that corresponding noncommutativity parameters are just the background fields of the fermionic T-dual theory (2.13)-(2.15).

4.1 Choice of the boundary conditions and canonical consistency procedure

Varying the Hamiltonian (3.19) we obtain

$$\delta H_c = \delta H_c^{(R)} - \left[\tilde{\gamma}_{\mu}^{(0)} \delta x^{\mu} + \frac{1}{4\kappa} J_{+\alpha} F^{\alpha\beta} \delta \eta_{\beta} + \frac{1}{4\kappa} \delta \bar{\eta}_{\alpha} F^{\alpha\beta} \bar{J}_{-\alpha} \right] \Big|_0^{\pi}, \quad (4.1)$$

where $\delta H_c^{(R)}$ is regular term, without derivatives of coordinates and momenta variations, and

$$\tilde{\gamma}_{\mu}^{(0)} = \Pi_{+\mu\nu} \star J_{-}^{\nu} + \Pi_{-\mu\nu} \star J_{+}^{\nu}. \quad (4.2)$$

Because the Hamiltonian is time translation generator it must have well defined functional derivatives with respect to the coordinates and momenta. Consequently, we get the boundary condition

$$\left[\tilde{\gamma}_{\mu}^{(0)} \delta x^{\mu} + \frac{1}{4\kappa} J_{+\alpha} F^{\alpha\beta} \delta \eta_{\beta} + \frac{1}{4\kappa} \delta \bar{\eta}_{\alpha} F^{\alpha\beta} \bar{J}_{-\alpha} \right] \Big|_0^{\pi} = 0. \quad (4.3)$$

We will choose Dirichlet boundary conditions (fixed string endpoints)

$$x^{\mu}|_0^{\pi} = const., \quad \eta_{\alpha}|_0^{\pi} = const., \quad \bar{\eta}_{\alpha}|_0^{\pi} = const., \quad (4.4)$$

which solve boundary condition (4.3). They can be expressed in more suitable form in terms of the currents

$$\begin{aligned} \gamma_{\mu}^{(0)}|_0^{\pi} &= 0, & \gamma_{\mu}^{(0)} &\equiv J_{+\mu} + J_{-\mu}, \\ \gamma_{\alpha}^{(0)}|_0^{\pi} &= 0, & \gamma_{\alpha}^{(0)} &\equiv J_{+\alpha} + J_{-\alpha}, \\ \bar{\gamma}_{\alpha}^{(0)}|_0^{\pi} &= 0, & \bar{\gamma}_{\alpha}^{(0)} &\equiv \bar{J}_{+\alpha} + \bar{J}_{-\alpha}. \end{aligned} \quad (4.5)$$

In fact, on the equations of motion for momenta π_μ , π_α and $\bar{\pi}_\alpha$ we have

$$J_{\pm\mu} = \kappa G_{\mu\nu} \partial_\pm x^\nu + \frac{1}{2} \bar{\Psi}_\mu^\alpha \partial_\pm \eta_\alpha + \frac{1}{2} \partial_\pm \bar{\eta}_\alpha \Psi_\mu^\alpha, \quad J_{\pm\alpha} = -\frac{1}{2} \partial_\pm \bar{\eta}_\alpha, \quad \bar{J}_{\pm\alpha} = \frac{1}{2} \partial_\pm \eta_\alpha, \quad (4.6)$$

which means that string endpoints velocities are zero

$$2\kappa G_{\mu\nu} \dot{x}^\nu = \gamma_\mu^{(0)} + \gamma_\alpha^{(0)} \Psi_\mu^\alpha - \bar{\Psi}_\mu^\alpha \bar{\gamma}_\alpha^{(0)}, \quad -\dot{\bar{\eta}}_\alpha = \gamma_\alpha^{(0)}, \quad \dot{\eta}_\alpha = \bar{\gamma}_\alpha^{(0)}. \quad (4.7)$$

Following method developed in Refs.[5, 6] we will consider the expressions $\gamma_A^{(0)} = (\gamma_\mu^{(0)}, \gamma_\alpha^{(0)}, \bar{\gamma}_\alpha^{(0)})$ as the canonical constraints. Applying Dirac consistency procedure we obtain infinite set of the constraints

$$\gamma_A^{(n)}|_0^\pi = 0, \quad \gamma_A^{(n)} \equiv \left\{ H_c, \gamma_A^{(n-1)} \right\}, \quad (n = 1, 2, 3, \dots) \quad (4.8)$$

where

$$\begin{aligned} \gamma_\mu^{(n)} &= (-1)^n \partial_\sigma^n J_{+\mu} + \partial_\sigma^n J_{-\mu}, \\ \gamma_\alpha^{(n)} &= (-1)^n \partial_\sigma^n J_{+\alpha} + \partial_\sigma^n J_{-\alpha}, \quad (n = 1, 2, 3, \dots) \\ \bar{\gamma}_\alpha^{(n)} &= (-1)^n \partial_\sigma^n \bar{J}_{+\alpha} + \partial_\sigma^n \bar{J}_{-\alpha}. \end{aligned} \quad (4.9)$$

With the help of the relation (3.20), using Taylor expansion

$$\Gamma_A(\sigma) = \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_A^{(n)}|_0, \quad [\Gamma_A = (\Gamma_\mu, \Gamma_\alpha, \bar{\Gamma}_\alpha)] \quad (4.10)$$

we rewrite these infinite sets of consistency conditions at $\sigma = 0$ in compact, σ dependent form

$$\Gamma_\mu(\sigma) = J_{+\mu}(-\sigma) + J_{-\mu}(\sigma), \quad (4.11)$$

$$\Gamma_\alpha(\sigma) = J_{+\alpha}(-\sigma) + J_{-\alpha}(\sigma), \quad (4.12)$$

$$\bar{\Gamma}_\alpha(\sigma) = \bar{J}_{+\alpha}(-\sigma) + \bar{J}_\alpha(\sigma). \quad (4.13)$$

In the similar way we can write the consistency conditions at $\sigma = \pi$. If we impose 2π periodicity of the canonical variables, the solution of the constraints at $\sigma = 0$ also solve the constraints at $\sigma = \pi$.

Because of the relation

$$\{H_c, \Gamma_A\} = \Gamma'_A \approx 0, \quad (4.14)$$

there are no other constraints in the theory and the consistency procedure is completed.

Using the algebra of the currents (3.13) we obtain the algebra of the constraints

$$\{\Gamma_A(\sigma), \Gamma_B(\bar{\sigma})\} = -4\kappa^* G_{AB} \delta', \quad (4.15)$$

and conclude that they are of the second class because the metric $^*G_{AB}$ defined in (3.8) is nonsingular for $\det^*G_{\mu\nu} \neq 0$ and $\det^*F_{\alpha\beta} \neq 0$.

In bosonic case the algebra of the constraints closes on bosonic T-dual fields. In the particular case the algebra of the constraints (4.15) closes on fermionic T-dual background fields (3.8).

4.2 Solution of the boundary conditions Γ_A

Let us first introduce new variables symmetric and antisymmetric under world-sheet parity transformation $\Omega : \sigma \rightarrow -\sigma$. For bosonic variables and fermionic momenta we use standard notation [6]

$$q^\mu(\sigma) = P_s x^\mu(\sigma), \quad \tilde{q}^\mu(\sigma) = P_a x^\mu(\sigma), \quad (4.16)$$

$$p_\mu(\sigma) = P_s \pi_\mu(\sigma), \quad \tilde{p}_\mu(\sigma) = P_a \pi_\mu(\sigma) \quad (4.17)$$

$$\tilde{p}_\alpha(\sigma) = P_a \pi_\alpha(\sigma), \quad \tilde{\tilde{p}}_\alpha(\sigma) = P_a \bar{\pi}_\alpha(\sigma), \quad (4.18)$$

while for fermionic coordinates we use subscript a

$$\theta_a^\alpha(\sigma) = P_a \theta^\alpha(\sigma), \quad \bar{\theta}_a^\alpha(\sigma) = P_a \bar{\theta}^\alpha(\sigma), \quad (4.19)$$

where the projectors on Ω even and odd parts are

$$P_s = \frac{1}{2}(1 + \Omega), \quad P_a = \frac{1}{2}(1 - \Omega). \quad (4.20)$$

Now we are ready to solve the constraint equations

$$\Gamma_\mu(\sigma) = 0, \quad \Gamma_\alpha(\sigma) = 0, \quad \bar{\Gamma}_\alpha(\sigma) = 0. \quad (4.21)$$

We obtain initial variables in terms of the effective ones

$$x^\mu(\sigma) = \tilde{q}^\mu(\sigma), \quad \pi_\mu = \tilde{p}_\mu - 2\kappa {}^*B_{\mu\nu} \tilde{q}'^\nu + \frac{\kappa}{2} ({}^*\bar{\Psi}_{\alpha\mu} \theta_a'^\alpha + \bar{\theta}_a'^\alpha {}^*\Psi_{\alpha\mu}), \quad (4.22)$$

$$\theta^\alpha(\sigma) = \theta_a^\alpha(\sigma), \quad \pi_\alpha = \tilde{p}_\alpha - \frac{\kappa}{8} \bar{\theta}_a'^\beta {}^*F_{\beta\alpha} + \frac{\kappa}{2} {}^*\bar{\Psi}_{\alpha\mu} \tilde{q}'^\mu, \quad (4.23)$$

$$\bar{\theta}^\alpha(\sigma) = \bar{\theta}_a^\alpha(\sigma), \quad \bar{\pi}_\alpha = \tilde{\bar{p}}_\alpha - \frac{\kappa}{8} {}^*F_{\alpha\beta} \theta_a'^\beta - \frac{\kappa}{2} {}^*\Psi_{\alpha\mu} \tilde{q}'^\mu, \quad (4.24)$$

where the fermionic dual background fields (with stars) are defined in (2.13)-(2.15). We can reexpress these solutions in terms of the initial background fields too

$$\begin{aligned} x^\mu(\sigma) &= \tilde{q}^\mu(\sigma), \quad \pi_\mu = \tilde{p}_\mu - 2\kappa B_{\mu\nu} \tilde{q}'^\nu - \frac{1}{2} \bar{\Psi}_\mu^\alpha (\eta'_a)_\alpha + \frac{1}{2} (\bar{\eta}'_a)_\alpha \Psi_\mu^\alpha, \\ \theta^\alpha(\sigma) &= \theta_a^\alpha(\sigma), \quad \pi_\alpha = \tilde{p}_\alpha - \frac{1}{2} (\bar{\eta}'_a)_\alpha, \\ \bar{\theta}^\alpha(\sigma) &= \bar{\theta}_a^\alpha(\sigma), \quad \bar{\pi}_\alpha = \tilde{\bar{p}}_\alpha - \frac{1}{2} (\eta'_a)_\alpha, \end{aligned} \quad (4.25)$$

where

$$(\eta_a)_\alpha \equiv 4\kappa (F^{-1})_{\alpha\beta} (\theta_a^\beta + \Psi_\mu^\beta \tilde{q}'^\mu), \quad (\bar{\eta}_a)_\alpha \equiv 4\kappa (\bar{\theta}_a^\beta + \bar{\Psi}_\mu^\beta \tilde{q}'^\mu) (F^{-1})_{\beta\alpha}, \quad (4.26)$$

are Ω odd projections of the variables (2.3). Note that, as a difference of all previous cases, our basic effective variables \tilde{q}^μ , \tilde{p}_μ , θ_a^α , \tilde{p}_α , $\bar{\theta}_a^\alpha$ and $\tilde{\bar{p}}_\alpha$ are Ω odd and the solution for momenta is nontrivial.

From basic Poisson bracket

$$\{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu \delta(\sigma - \bar{\sigma}), \quad (4.27)$$

we obtain the corresponding one in Ω odd subspace

$$\{\tilde{q}^\mu(\sigma), \tilde{p}_\nu(\bar{\sigma})\} = 2\delta^\mu{}_\nu \delta_a(\sigma, \bar{\sigma}), \quad (4.28)$$

where

$$\delta_a(\sigma, \bar{\sigma}) = \frac{1}{2} [\delta(\sigma - \bar{\sigma}) - \delta(\sigma + \bar{\sigma})], \quad (4.29)$$

is antisymmetric delta function. The factor 2 in front of antisymmetric delta function comes from the fact that Ω -odd functions on the interval $[-\pi, \pi]$, \tilde{q}^μ and \tilde{p}_ν , are restricted on the interval $[0, \pi]$ (see [17]).

Similarly, using basic Poisson algebra of fermionic variables

$$\{\theta^\alpha(\sigma), \pi_\beta(\bar{\sigma})\} = \{\bar{\theta}^\alpha(\sigma), \bar{\pi}_\beta(\bar{\sigma})\} = -\delta^\alpha{}_\beta \delta(\sigma - \bar{\sigma}), \quad (4.30)$$

we have

$$\{\theta_a^\alpha(\sigma), \tilde{p}_\beta(\bar{\sigma})\} = -2\delta^\alpha{}_\beta \delta_a(\sigma, \bar{\sigma}), \quad \{\bar{\theta}_a^\alpha(\sigma), \tilde{\bar{p}}_\beta(\bar{\sigma})\} = -2\delta^\alpha{}_\beta \delta_a(\sigma, \bar{\sigma}). \quad (4.31)$$

The momenta \tilde{p}_μ , \tilde{p}_α and $\tilde{\bar{p}}_\alpha$ are canonically conjugated to the coordinates \tilde{q}^μ , θ_a^α and $\bar{\theta}_a^\alpha$, respectively, in Ω odd subspace.

5 Momenta noncommutativity relations

When the Neumann boundary conditions have been used [10, 11], the solution for the super momenta was trivial while the solution for the super coordinates depended not only on the effective coordinates but also on the effective momenta. This was a source of the coordinate noncommutativity which corresponded to the bosonic T-duality. In the present case, with Dirichlet boundary conditions, the solution for the super coordinates is trivial while the solution for the super momenta depends not only on the effective momenta but also on the effective coordinates. This is a source of momenta noncommutativity which will correspond to the fermionic T-duality.

Instead to calculate Dirac brackets in the initial phase space associated with constraints Γ_A , we will calculate the equivalent brackets in the reduced phase space. We will put the subscript D to distinguish them from Poisson ones of initial phase space. With the help of the solution (4.22)-(4.24) we find that all supercoordinates are commutative, while the D brackets of momenta have a form

$$\{\pi_\mu(\sigma), \pi_\nu(\bar{\sigma})\}_D = 4\kappa {}^*B_{\mu\nu} \partial_\sigma \delta(\sigma + \bar{\sigma}), \quad (5.1)$$

$$\{\pi_\mu(\sigma), \pi_\alpha(\bar{\sigma})\}_D = \kappa {}^*\bar{\Psi}_{\mu\alpha} \partial_\sigma \delta(\sigma + \bar{\sigma}), \quad (5.2)$$

$$\{\pi_\mu(\sigma), \bar{\pi}_\alpha(\bar{\sigma})\}_D = -\kappa {}^*\Psi_{\alpha\mu} \partial_\sigma \delta(\sigma + \bar{\sigma}), \quad (5.3)$$

$$\{\pi_\alpha(\sigma), \bar{\pi}_\beta(\bar{\sigma})\}_D = -\frac{\kappa}{4} {}^*F_{\beta\alpha} \partial_\sigma \delta(\sigma + \bar{\sigma}), \quad (5.4)$$

$$\{\pi_\alpha(\sigma), \pi_\beta(\bar{\sigma})\}_D = \{\bar{\pi}_\alpha(\sigma), \bar{\pi}_\beta(\bar{\sigma})\}_D = 0. \quad (5.5)$$

If we define the variables

$$\Pi_\mu(\sigma) = \int_0^\sigma d\sigma_1 \pi_\mu(\sigma_1), \quad \Pi_\alpha = \int_0^\sigma d\sigma_1 \pi_\alpha(\sigma_1), \quad \bar{\Pi}_\alpha = \int_0^\sigma d\sigma_1 \bar{\pi}_\alpha(\sigma_1), \quad (5.6)$$

the noncommutativity relations turn to the standard form

$$\{\Pi_\mu(\sigma), \Pi_\nu(\bar{\sigma})\}_D = 4 \kappa {}^*B_{\mu\nu} \theta(\sigma + \bar{\sigma}), \quad (5.7)$$

$$\{\Pi_\mu(\sigma), \Pi_\alpha(\bar{\sigma})\}_D = \kappa {}^*\bar{\Psi}_{\mu\alpha} \theta(\sigma + \bar{\sigma}), \quad (5.8)$$

$$\{\Pi_\mu(\sigma), \bar{\Pi}_\alpha(\bar{\sigma})\}_D = -\kappa {}^*\Psi_{\alpha\mu} \theta(\sigma + \bar{\sigma}), \quad (5.9)$$

$$\{\Pi_\alpha(\sigma), \bar{\Pi}_\beta(\bar{\sigma})\}_D = -\frac{\kappa}{4} {}^*F_{\beta\alpha} \theta(\sigma + \bar{\sigma}), \quad (5.10)$$

$$\{\Pi_\alpha(\sigma), \Pi_\beta(\bar{\sigma})\}_D = \{\bar{\Pi}_\alpha(\sigma), \bar{\Pi}_\beta(\bar{\sigma})\}_D = 0, \quad (5.11)$$

where

$$\theta(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1/2 & \text{if } 0 < x < 2\pi \\ 1 & \text{if } x = 2\pi \end{cases} \quad (5.12)$$

Separating the mean value of momenta

$$\Pi_A(\sigma) = \Pi_A^{mv} + \mathcal{P}_A(\sigma), \quad \Pi_A^{mv} = \frac{1}{\pi} \int_0^\pi d\sigma \Pi_A(\sigma),$$

we obtain that only integrals of the momenta at the string endpoints are noncommutative

$$\{\mathcal{P}_\mu(\sigma), \mathcal{P}_\nu(\bar{\sigma})\}_D = \Theta_{\mu\nu} \Delta(\sigma + \bar{\sigma}), \quad (5.13)$$

$$\{\mathcal{P}_\mu(\sigma), \mathcal{P}_\alpha(\bar{\sigma})\}_D = \bar{\Theta}_{\mu\alpha} \Delta(\sigma + \bar{\sigma}), \quad (5.14)$$

$$\{\mathcal{P}_\mu(\sigma), \bar{\mathcal{P}}_\alpha(\bar{\sigma})\}_D = \Theta_{\alpha\mu} \Delta(\sigma + \bar{\sigma}), \quad (5.15)$$

$$\{\mathcal{P}_\alpha(\sigma), \bar{\mathcal{P}}_\beta(\bar{\sigma})\}_D = \Theta_{\alpha\beta} \Delta(\sigma + \bar{\sigma}), \quad (5.16)$$

$$\{\mathcal{P}_\alpha(\sigma), \mathcal{P}_\beta(\bar{\sigma})\}_D = \{\bar{\mathcal{P}}_\alpha(\sigma), \bar{\mathcal{P}}_\beta(\bar{\sigma})\}_D = 0, \quad (5.17)$$

where the noncommutativity parameters are defined as

$$\Theta_{\mu\nu} = 2\kappa {}^*B_{\mu\nu}, \quad \bar{\Theta}_{\mu\alpha} = \frac{\kappa}{2} {}^*\bar{\Psi}_{\mu\alpha}, \quad \Theta_{\alpha\mu} = -\frac{\kappa}{2} {}^*\Psi_{\alpha\mu}, \quad \Theta_{\alpha\beta} = -\frac{\kappa}{8} {}^*F_{\beta\alpha}, \quad (5.18)$$

and

$$\Delta(x) = 2\theta(x) - 1 = \begin{cases} -1 & \text{if } x = 0 \\ 0 & \text{if } 0 < x < 2\pi \\ 1 & \text{if } x = 2\pi \end{cases} \quad (5.19)$$

Therefore, all background fields of the fermionic T-dual theory (2.13)-(2.15), except ${}^*G_{\mu\nu}$, appear as noncommutativity parameters for the solution of boundary conditions (4.4).

6 Concluding remarks

In the present article we considered the relationship between fermionic T-duality and noncommutativity in type IIB superstring theory. We used the pure spinor formulation of the theory keeping all terms up to the quadratic ones and neglecting ghost terms. Our goal was to find such solution of the boundary conditions which will produce fermionic T-dual fields as noncommutativity parameters.

First, we performed fermionic T-duality in the way described in Refs.[8]. Comparing initial and dualized theory, we found the expressions for fermionic T-dual background fields.

Varying the canonical Hamiltonian and demanding that it has well defined functional derivatives with respect to the coordinates and momenta, we obtain the boundary condition (4.3). In order to satisfy them, for all supercoordinates we chose Dirichlet boundary conditions. Treating these conditions as canonical constraints and reexpressing them in terms of useful introduced currents we were able to examine consistency of the constraints. For nonsingular dual metric ${}^*G_{\mu\nu}$ and nonsingular dual R-R field strength ${}^*F_{\alpha\beta}$ all constraints are of the second class.

Instead to use Dirac brackets we solved the second class constraints. We took Ω odd parts of canonical variables as independent effective variables, and expressed the Ω even ones in terms of them. We found that the solution of supercoordinates was trivial, because they depended only on its Ω odd projections. The solutions for supermomenta depend both on effective supercoordinates and effective supermomenta. So, as a difference of the previous investigations [5, 10, 11, 6, 14, 15], here supercoordinates are commutative while integrals of supermomenta are noncommutative. Similar as in previous investigations, noncommutativity appears only at the string endpoints, and not in the string interior. Noncommutativity parameters at $\sigma = 0$ and $\sigma = \pi$ have opposite signs.

Let us comment relation between fermionic T-dual background fields defined in (2.13)-(2.15) with noncommutative parameters corresponding to the Dirichlet boundary conditions (4.4). All noncommutativity parameters, up to the some constant multipliers, are equal to the fermionic T-dual fields. Because noncommutativity relations close on $\Delta(\sigma + \bar{\sigma})$ which is symmetric under $\sigma \leftrightarrow \bar{\sigma}$, the noncommutativity parameter symmetric in space-time indices is absent. Therefore, only T-dual metric tensor ${}^*G_{\mu\nu}$ does not appear as noncommutativity parameter. As well as in the previous cases, dual fields appear in the algebra of constraints (4.15). Because here the algebra closes on $\partial_\sigma \delta(\sigma - \bar{\sigma})$ which is antisymmetric under $\sigma \leftrightarrow \bar{\sigma}$, the background field antisymmetric in space-time indices is absent. So, the D-brackets of σ -dependent constraints Γ_A close on all T-dual background fields except ${}^*B_{\mu\nu}$.

There is analogy of the obtained result with that of Ref.[11]. We present that in the following table.

Bosonic T-duality	Fermionic T-duality
Neumann boundary conditions for bosonic coordinates	Dirichlet boundary conditions for all coordinates
Ω even effective supercoordinates	Ω odd effective supercoordinates
Supercoordinates noncommutativity	Supermomenta noncommutativity

Table 1: Analogy between bosonic T-duality and coordinates noncommutativity with fermionic T-duality and momenta noncommutativity

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